IONIZATION MODEL CORIMIA WITH 3+1 INTERVAL APPROXIMATION FOR COSMIC RAYS

Peter I. Y. I. Velinov, Simeon Asenovski, Radostin Pavlov, Lachezar Mateev

Space Research and Technology Institute – Bulgarian Academy of Sciences
e-mail: peteriyvelinov@gmail.com

Keywords: *Cosmic Ray Ionization, CORIMIA Model, Three-Interval Approximation, Energy Loss Function, Proton Ionization, Atmospheric Ionization*

Abstract: *The CORIMIA ionization model is used to describe the process of ionization in the Earth's atmosphere caused by cosmic ray particles. In this study, a three plus one interval approximation of the ionization loss function dE/dh is presented for cosmic rays. The approach extends the previous two interval model by introducing a third interval for higher particle energies (Ek > 5 GeV), improving the accuracy of ionization rate calculations in the upper atmospheric regions. Analytical expressions are derived for the energy loss laws and the corresponding atmospheric cut-off parameters in each interval. The obtained formulation allows a more detailed estimation of the ionization yield and contributes to a better representation of the energy dependent behavior of cosmic rays.*

ЙОНИЗАЦИОНЕН МОДЕЛ CORIMIA С 3 + 1 ИНТЕРВАЛНА АПРОКСИМАЦИЯ ЗА КОСМИЧЕСКИ ЛЪЧИ

Петър И. Велинов, Симеон Асеновски, Радостин Павлов, Лъчезар Матев

Институт за космически изследвания и технологии – Българска академия на науките
e-mail: peteriyvelinov@gmail.com

Ключови думи: *йонизация от космически лъчи, Модел CORIMIA, три-интервална аппроксимация, функция на енергийните загуби, йонизация от протони, атмосферна йонизация*

Резюме: *Йонизационният Модел CORIMIA се използва за описание на процеса на йонизация в земната атмосфера, предизвикан от космическите лъчи. В настоящото изследване е представена три плюс един интервална аппроксимация на функцията на йонизационните загуби dE/dh за космически лъчи протони със заряд Z = 1. Подходът разширява предишния дву-интервален модел чрез въвеждане на трети интервал за по-високи енергии на частиците (Ek > 5 GeV), като по този начин се повишава точността при изчисленията на скоростта на йонизация в горните атмосферни слоеве. Изведени са аналитични изрази за законите на енергийните загуби и съответните атмосферни прагове на отрязване за всеки интервал. Получената формулировка позволява по-прецизна оценка на йонизационния добив и по-добро представяне на енергийнозависимото поведение на космическите лъчи.*

Introduction of three interval approximation of the ionization loss function for particles with z > 1

The equation describing the ionization losses in the case of the three-interval approximation for particles with charge Z has the form:

(1)
$$-\frac{1}{\rho}\frac{dE}{dh} = \begin{cases} 1613.3E^{1/4} & 0.15 \le E \le E_a \quad \text{INTERVAL } 1^* \\ 242Z^2E^{-3/4} & E_a < E \le 600 \text{ MeV} \quad \text{INTERVAL } 1 \\ 2Z^2 & 600 < E \le 5000 \text{ MeV} \quad \text{INTERVAL } 2 \\ 0.7Z^2E^{0.123} & 5 < E \le 5000 \text{ GeV} \quad \text{INTERVAL } 3. \end{cases}$$

In addition to the three intervals, Interval $1*$ is also introduced, which describes the decrease in the charge of the incoming particle. The upper limit of this interval is given by the expression:

(2) $$E_a = 0.15 \times Z^2.$$

The first two intervals, as well as the charge reduction interval in equation (1), are identical to the two-interval approximation, i.e. we have no change in the slope of the curve describing the ionization losses. Therefore, in describing the model for the three-interval approximation, we will use some of the formulas of the previous case.

The first step that needs to be taken is to derive the energy loss laws for the individual intervals. For Interval $1*$, 1 and 2 we have respectively:

(3) $$E_{1^*}(h) = \left(E_k^{3/4} - \frac{1210}{A}\tilde{h}\right)^{4/3}.$$

(4) $$E_1(h) = \left(E_k^{7/4} - \frac{423.75Z^2}{A}\tilde{h}\right)^{4/7}.$$

(5) $$E_2(h) = E_k - \frac{2Z^2}{A}\tilde{h}.$$

The energy loss law describing Interval 3 will be found using the amount of substance passed:

(6) $$\tilde{h} = \tilde{h}_3 = \frac{A}{0.7Z^2}\int_{E(h)}^{E_k}\frac{dE}{E^{0.123}} = \frac{A}{0.61Z^2}\left(E_k^{0.877} - E_3^{0.877}(h)\right).$$

From equation (6) we obtain:

(7) $$E_3(h) = \left(E_k^{0.877} - \frac{0.61Z^2}{A}\tilde{h}\right)^{1.14}.$$

The atmospheric cutoff thresholds for the individual intervals should be expressed. Using the calculations we have:

(8) $$E_{A1^*}(h) = \left(0.15^{3/4} + \frac{1210}{A}\tilde{h}\right)^{4/3} \leq E_a.$$

(9) $$E_{A1}(h) = \left(423.74\frac{Z^2}{A}\tilde{h} - 0.35Z^2(E_a^{3/4} - 0.15^{3/4}) + E_a^{7/4}\right)^{4/7}.$$

(10) $$E_{A2}(h) = 2\frac{Z^2}{A}\tilde{h} - 0.002Z^2(E_a^{3/4} - 0.15^{3/4}) - 0.005(600^{7/4} - E_a^{7/4}) + 600.$$

The atmospheric cutoff threshold for Interval 3 can be expressed as follows:

(11) $$\begin{aligned}\tilde{h} = \tilde{h}_{1^*} + \tilde{h}_1 + \tilde{h}_2 + \tilde{h}_3 &= \frac{A}{1613.3}\int_{E_{0.15}}^{E_a}\frac{dE}{E^{1/4}} + \frac{A}{242Z^2}\int_{E_a}^{600}\frac{dE}{E^{-3/4}} + \\ &+ \frac{A}{2Z^2}\int_{600}^{5000}dE + \frac{A}{0.7Z^2}\int_{5000}^{E_{A3}(h)}\frac{dE}{E^{0.123}}dE = \\ &= \frac{A}{1210}\left(E_a^{3/4} - 0.15^{3/4}\right) + \frac{A}{423.75Z^2}\left(600^{7/4} - E_a^{7/4}\right) + \\ &+ \frac{A}{2Z^2}(5000 - 600) + \frac{A}{0.61Z^2}\left(E_{A3}^{0.877}(h) - 5000^{0.877}\right).\end{aligned}$$

from where we get:

(12) $$\begin{aligned}E_{A3}(h) = \Big(&\frac{0.61Z^2}{A}\tilde{h} + 5000^{0.877} - 5\times10^{-4}Z^2\left(E_a^{3/4} - 0.15^{3/4}\right) - \\ &- 0.0014\left(600^{7/4} - E_a^{7/4}\right) - 0.3\,(5000 - 600)\Big)^{1.14}.\end{aligned}$$

The significance of the atmospheric cutoff thresholds for the different intervals is important for determining the effective cutoff threshold.

An important step in describing the model is to derive the energy loss laws for cases where the energy of the incoming particle crosses one or more interval boundaries. When the energy of a particle with initial energy in Interval 1 crosses the boundary Ea, the energy loss law is given by the expression:

(13) $$E_{11^*}(h) = \left(\frac{2.85}{Z^2}(E_k^{7/4} - E_a^{7/4}) - \frac{1210}{A}\tilde{h} + E_a^{3/4}\right)^{4/3}.$$

In the case where the energy of the incoming particle is in Interval 2 and crosses the 600 MeV limit, we have (equation 4.45)):

(14) $$E_{21}(h) = \left(211.88(E_k - 600) - \frac{423.75Z^2}{A}\tilde{h} + 600^{7/4}\right)^{4/7}.$$

The law of energy loss when crossing the 5000 MeV limit can similarly be derived using the amount of substance passed:

(15) $$\tilde{h} = \tilde{h}_2 + \tilde{h}_3 = \frac{A}{2Z^2}\int_{E_{32}}^{5000} dE + \frac{A}{0.7Z^2}\int_{5000}^{E_k}\frac{dE}{E^{0.123}}dE =$$

$$= \frac{A}{2Z^2}\left(5000 - E_{32}(h)\right) + \frac{A}{0.61Z^2}\left(E_k^{0.877} - 5000^{0.877}\right).$$

Finally for E32(h) we obtain:

(16) $$E_{32}(h) = 5000 - \frac{2Z^2}{A}\tilde{h} + 2.86\left(E_k^{0.877} - 5000^{0.877}\right).$$

When the energy of the particle crosses two boundaries, we have two possibilities. If the initial energy of the particle is in Interval 2 and when passing through the atmosphere the energy crosses the boundaries of 600 MeV and Ea, the energy loss law is given by equation:

(17) $$E_{21^*}(h) = \left(E_a^{3/4} + \frac{2.85}{Z^2}(600^{7/4} - E_a^{7/4}) + \frac{605}{Z^2}(E_k - 600) - \frac{1210}{A}\tilde{h}\right)^{4/3}.$$

The second possibility is when the initial energy of the incoming particle Ek is in Interval 3, and subsequently the kinetic and energy crosses the boundaries of 5000 MeV and 600 MeV. For the amount of substance passed in this case we have:

(18) $$\tilde{h} = \tilde{h}_1 + \tilde{h}_2 + \tilde{h}_3 = \frac{A}{242Z^2}\int_{E_{31}}^{600}\frac{dE}{E^{-3/4}} + \frac{A}{2Z^2}\int_{600}^{5000} dE + \frac{A}{0.7Z^2}\int_{5000}^{E_k}\frac{dE}{E^{0.123}}dE =$$

$$= \frac{A}{423.75Z^2}\left(600^{7/4} - E_{31}^{7/4}\right) + \frac{A}{2Z^2}\left(5000 - 600\right) + \frac{A}{0.61Z^2}\left(E_k^{0.877} - 5000^{0.877}\right).$$

From equation (18) we can express the energy loss law E31(h):

(19) $$E_{31}(h) = \left(600^{7/4} + 211.88\left(5000 - 600\right) + 694.67\left(E_k^{0.877} - 5000^{0.877}\right) - \frac{423.11Z^2}{A}\tilde{h}\right)^{4/7}.$$

There is one possibility where the kinetic energy of the incoming particle can cross the boundaries of all intervals. In this case, we can write:

(20) $$\tilde{h} = \tilde{h}_{1^*} + \tilde{h}_1 + \tilde{h}_2 + \tilde{h}_3 = \frac{A}{1613.3}\int_{E_{31^*}}^{E_a}\frac{dE}{E^{1/4}} + \frac{A}{242Z^2}\int_{E_a}^{600}\frac{dE}{E^{-3/4}} +$$

$$+\frac{A}{2Z^2}\int_{600}^{5000} dE + \frac{A}{0.7Z^2}\int_{5000}^{E_k}\frac{dE}{E^{0.123}}dE =$$

$$= \frac{A}{1210}\left(E_a^{3/4} - E_{31^*}^{3/4}\right) + \frac{A}{423.75Z^2}\left(600^{7/4} - E_a^{7/4}\right) +$$

$$+\frac{A}{2Z^2}\left(5000 - 600\right) + \frac{A}{0.61Z^2}\left(E_k^{0.877} - 5000^{0.877}\right).$$

The energy loss law in the case of crossing 3 interval boundaries has the form:

(21) $$E_{31^*}(h) = \left(E_a^{3/4} - \frac{1210}{A}\tilde{h} + \frac{2.85}{Z^2}\left(600^{7/4} - E_a^{7/4}\right) + \right.$$

$$\left. + \frac{605}{Z^2}(5000 - 600) + \frac{1983.6}{Z^2}\left(E_k^{0.877} - 5000^{0.877}\right)\right)^{4/3}.$$

To express the explicit form of the model for calculating the electron production, it is necessary to find the ionization loss function for the individual intervals and in cases where there is a crossing of energy boundaries by the kinetic energy of the incoming particle. For this purpose, we will use the laws for energy losses and as well as the laws for energy losses in cases of crossing of boundaries of intervals:

(22) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{1^*} = 1613.3E_{1^*}^{1/4} = 1613.3\left(E_k^{3/4} - \frac{1210}{A}\tilde{h}\right)^{1/3},$$

(23) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{1} = 242Z^2E_1^{-3/4} = 242Z^2\left(E_k^{7/4} - \frac{423.75Z^2}{A}\tilde{h}\right)^{-3/7},$$

(24) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{2} = \frac{2Z^2}{A},$$

(25) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{11^*} = 1613.3E_{11^*}^{1/4} = 1613.3\left(\frac{2.85}{Z^2}(E_k^{7/4} - E_a^{7/4}) - \frac{1210}{A}\tilde{h} + E_a^{3/4}\right)^{1/3},$$

(26) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{21} = 242Z^2E_{21}^{-3/4} = 242Z^2\left(211.88(E_k - 600) - \frac{423.75Z^2}{A}\tilde{h} + 600^{7/4}\right)^{-3/7},$$

(27) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{21^*} = 1613.3E_{21^*}^{1/4} = 1613.3\left(\frac{2.85}{Z^2}(600^{7/4} - E_a^{7/4}) - \frac{605}{Z^2}(E_k - 600) - \frac{1210}{A}\tilde{h} + E_a^{3/4}\right)^{1/3},$$

(28) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{3} = 0.7Z^2E_3^{0.123}(h) = 0.7Z^2\left(E_k^{0.877} - \frac{0.61Z^2}{A}\tilde{h}\right)^{0.14},$$

(29) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{32} = \frac{2Z^2}{A},$$

(30) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{31} = 242Z^2E_{31}^{-3/4} =$$

$$= 242Z^2\left(600^{7/4} + 211.88\,(5000 - 600) + 694.67\left(E_k^{0.877} - 5000^{0.877}\right) - \frac{423.11Z^2}{A}\tilde{h}\right)^{-3/7},$$

(31) $$\frac{1}{\rho}\left(\frac{dE}{dh}\right)_{31^*} = 1613.3E_{31^*}^{1/4} = 1613.3\left(E_a^{3/4} - \frac{1210}{A}\tilde{h} + \frac{2.85}{Z^2}\left(600^{7/4} - E_a^{7/4}\right) + \right.$$

$$\left. + \frac{605}{Z^2}(5000 - 600) + \frac{1983.6}{Z^2}\left(E_k^{0.877} - 5000^{0.877}\right)\right)^{1/3}.$$

Before we can express the electron production, we need to find the energies of infinity (the initial kinetic energy before the particle enters the Earth's atmosphere) for the boundaries of the individual intervals. Again, we will use the amount of matter passed. For the boundary Ea = 0.15 × Z2, three cases are possible depending on the magnitude of the energy of infinity. The first case is when the energy of infinity is in Interval 1. Then we have:

(32) $$\tilde{h} = \tilde{h}_1 = \frac{A}{242Z^2}\int_{E_a}^{E_{a,1}}\frac{dE}{E^{-3/4}} = \frac{A}{423.75Z^2}\left(E_{a,1}^{7/4} - E_a^{7/4}\right) \Rightarrow$$

(33) $$E_{a,1} = \left(E_a^{7/4} + \frac{423.75Z^2}{A}\tilde{h}\right)^{4/7}.$$

When the particle's energy at infinity is in Interval 2 we get:

(34) $$\tilde{h} = \tilde{h}_1 + \tilde{h}_2 = \frac{A}{242Z^2}\int_{E_a}^{600}\frac{dE}{E^{-3/4}} + \frac{A}{2Z^2}\int_{600}^{E_{a,2}} dE =$$

$$= \frac{A}{423.75Z^2}\left(600^{7/4} - E_a^{7/4}\right) + \frac{A}{2Z^2}\left(E_{a,2} - 600\right) \Rightarrow$$

(35) $$E_{a,2} = 600 + \frac{2Z^2}{A}\tilde{h} - 211.88\left(600^{7/4} - E_a^{7/4}\right).$$

For the last case, when the energy of infinity for the boundary Ea is in Interval 3, we have:

(36) $$\tilde{h} = \tilde{h}_1 + \tilde{h}_2 + \tilde{h}_3 = \frac{A}{242Z^2}\int_{E_a}^{600}\frac{dE}{E^{-3/4}} + \frac{A}{2Z^2}\int_{600}^{5000} dE + \frac{A}{0.7Z^2}\int_{5000}^{E_{a,3}}\frac{dE}{E^{0.123}} =$$

$$= \frac{A}{423.75Z^2}\left(600^{7/4} - E_a^{7/4}\right) + \frac{A}{2Z^2}\left(5000 - 600\right) + \frac{A}{0.61Z^2}\left(E_{a,3}^{0.877} - 5000^{0.877}\right) \Rightarrow$$

(37) $$E_{a,3} = \left(5000^{0.877} + \frac{0.61Z^2}{A}\tilde{h} - 0.0014\left(600^{7/4} - E_a^{7/4}\right) - 0.305(5000 - 600)\right)^{1.14}$$

There are two possible cases for the energy of infinity at the 600 MeV limit. The first is when the energy of infinity is in Interval 2, in which case we get:

(38) $$\tilde{h} = \tilde{h}_2 = \frac{A}{2Z^2}\int_{600}^{E_{600,2}} dE = \frac{A}{2Z^2}\left(E_{600,2} - 600\right) \Rightarrow$$

(39) $$E_{600,2} = 600 + \frac{2Z^2}{A}\tilde{h}.$$

The second case is when the energy of infinity is in Interval 3:

(40) $$\tilde{h} = \tilde{h}_2 + \tilde{h}_3 = \frac{A}{2Z^2}\int_{600}^{5000} dE + \frac{A}{0.7Z^2}\int_{5000}^{E_{600,3}}\frac{dE}{E^{0.123}} =$$

$$= \frac{A}{2Z^2}\left(5000 - 600\right) + \frac{A}{0.61Z^2}\left(E_{600,3}^{0.877} - 5000^{0.877}\right) \Rightarrow$$

(41) $$E_{600,3} = \left(5000^{0.877} + \frac{0.61Z^2}{A}\tilde{h} - 0.305(5000 - 600)\right)^{1.14}.$$

For the 5000 MeV limit, the energy at infinity can only be in Interval 3, in which case we have:

(42) $$\tilde{h} = \tilde{h}_3 = \frac{A}{0.7Z^2}\int_{5000}^{E_{5000,3}}\frac{dE}{E^{0.123}} = \frac{A}{0.61Z^2}\left(E_{5000,3}^{0.877} - 5000^{0.877}\right) \Rightarrow$$

(43) $$E_{5000,3} = \left(5000^{0.877} + \frac{0.61Z^2}{A}\tilde{h}\right)^{1.14}.$$

After the calculations, we can construct the model for calculating the electron production due to the passage of charged cosmic rays with charge Z in a three-interval approximation of the ionization loss function. Depending on the size of the effective cutoff threshold Eeff, as well as the energies of infinity for the individual boundaries, several cases are possible for the electron production q(h).

Conclusions

The 3+1 interval approximation developed within the CORIMIA ionization model represents a significant refinement in the analytical treatment of cosmic ray–induced ionization. Compared with the earlier two-interval formulation, the inclusion of a high-energy domain (above 5 GeV) provides:

- improved continuity of the ionization loss function across energy boundaries,
- more realistic representation of relativistic energy losses,
- and enhanced accuracy of the ion production rate in the upper atmosphere.

The analytical expressions derived for Ei(h) and Eij(h) yield compact formulas for the atmospheric cut-off energies and penetration depths.

References:

1. Bazilevskaya, G.A., Usoskin, I.G., Flückiger, E.O. et al. (2008). Cosmic ray induced ion production in the atmosphere. Space Science Reviews, 137, 149–173.
2. Usoskin, I.G., Kovaltsov, G.A. (2006). Cosmic ray induced ionization in the atmosphere: Full modeling and practical applications. Journal of Geophysical Research, 111, D21206.
3. Velinov, P.I.Y., Asenovski, S., Mateev, L., Mishev, A. (2013). Improved COsmic Ray Ionization Model for Atmosphere and Ionosphere (CORIMIA) with account of Monte Carlo Simulations. Journal of Physics: Conference Series, 409, 012212. https://doi.org/10.1088/1742-6596/409/1/012212
4. Velinov, P.I.Y., Asenovski, S., Mateev, L. (2013). Ionization of Solar Cosmic Rays in Ionosphere and Middle Atmosphere Simulated by CORIMIA Programme. Comptes Rendus de l'Académie Bulgare des Sciences, 66(2), 235–242.
5. Moraal, H., McCracken, K.G. (2012). The time structure of ground level enhancements in solar cycle 23. Space Science Reviews, 171, 85–95.
6. O'Brien, K. Cosmic-ray propagation in the atmosphere. Nuov Cim A 3, 521–547 (1971). https://doi.org/10.1007/BF02823324
7. Bethe, H. (1930). Zur Theorie des Durchgangs schneller Korpuskularstrahlen durch Materie. Annalen der Physik, 397, 325–400.
8. Velinov, P.I.Y., Asenovski, S., Mateev, L., Mishev, A., Tonev, P.T. (2013). Numerical calculation of cosmic ray ionization rate profiles in the middle atmosphere and lower ionosphere with relation to characteristic energy intervals. Acta Geophysica, 61, 494–509.